

**National Institute of Technology, Agartala**  
**Department of Mathematics**



**Syllabus for**  
**Master of Science**  
**in**  
**Mathematics and Computing**

### Semester-I

Sl. No.	Name of the Subject	Marks	Teaching Scheme			Credit
			Hours per Week			
			L	Tu.	Pr.	
1.	Discrete Mathematics	100	3	1	0	4
2.	Linear Algebra	100	3	1	0	4
3.	Real Analysis	100	3	1	0	4
4.	Abstract Algebra	100	3	1	0	4
5.	Computer Programming in C++, Matlab/Mathematica (Theory)	50	2	0	0	2
6.	Computer Programming in C++, Matlab/Mathematica (Lab)	50	0	0	3	2
<b>Total Marks = 500</b>			<b>Total Credit = 20</b>			

### Semester-II

Sl. No.	Name of the Subject	Marks	Teaching Scheme			Credit
			Hours per Week			
			L	Tu.	Pr.	
1.	Topology	100	3	1	0	4
2.	Ordinary Differential Equation	100	3	1	0	4
3.	Complex Analysis	100	3	1	0	4
4.	Statistical inference and Stochastic Processes	100	3	1	0	4
5.	Numerical Methods and Scientific Computing	50	2	0	0	2
6.	Numerical Methods Lab	50	0	0	3	2
<b>Total Marks = 500</b>			<b>Total Credit = 20</b>			

### Semester-III

Sl. No.	Name of the Subject	Marks	Teaching Scheme			Credit
			Hours per Week			
			L	Tu.	Pr.	
1.	Mathematical Methods	100	3	1	0	4
2.	Partial Differential Equation	100	3	1	0	4
3.	Measure Theory	100	3	1	0	4
4.	* Elective – I	100	3	1	0	4
5.	* Elective – II	100	3	1	0	4
<b>Total Marks = 500</b>			<b>Total Credit = 20</b>			

### Semester-IV

<b>##Option – 1</b>						
Sl. No.	Name of the Subject	Marks	Teaching Scheme			Credit
			Hours per Week			
			L	Tu.	Pr.	
1.	# Elective – III	100	3	1	0	4
2.	#Elective – IV	100	3	1	0	4
3.	#Elective – V	100	3	1	0	4
4.	Project And Seminar	100	0	5	0	5
5.	Viva	100	0	3	0	3
<b>##Option – 2</b>						
6.	Industrial Project	300	0	12	0	12
7.	Project Seminar	100	0	5	0	5
8.	Comprehensive Viva	100	0	3	0	3
<b>Total Marks = 500</b>			<b>Total Credit = 20</b>			

**## Student of M.Sc Mathematics and Computing can choose either Option – 1 or Option – 2 in IV<sup>th</sup> Semester**

**Total Credit - 80**

**\*Elective – I and \*Elective – II**

<b>Subject Code</b>	<b>Name of the Subject</b>
	<b>Data Structure and Algorithm</b>
	<b>Operation Research – I</b>
	<b>Differential Geometry</b>
	<b>Number Theory and Cryptography</b>
	<b>Soft Computing – I</b>
	<b>Mathematical Modelling and Simulation – I</b>

**\* Subject to the availability of the faculty any two elective papers will be offered**

**#Elective – III, #Elective – IV and #Elective – V**

<b>Subject Code</b>	<b>Name of the Subject</b>
	<b>Advanced Numerical Analysis</b>
	<b>Operation Research – II</b>
	<b>Mechanics ( include Elasticity)</b>
	<b>Fluid Mechanics</b>
	<b>Soft Computing – II</b>
	<b>Mathematical Modelling and Simulation – II</b>
	<b>Graph Theory</b>
	<b>Theory of Computation</b>
	<b>Advanced Topology</b>
	<b>Functional Analysis</b>
	<b>Fuzzy Mathematics</b>
	<b>Decision Theory &amp; Computational Statistics</b>
	<b>Financial Mathematics</b>

**#Subject to the availability of the faculty any three elective papers will be offered**

# *Detailed Syllabus*

## M. Sc. 1<sup>st</sup> Semester

[Compulsory]

### Discrete Mathematics

#### Set Theory

Sets and classes, relations and functions, recursive definitions, posets, Zorn's lemma, cardinal and ordinal numbers.

#### Logic

Propositional and predicate calculus, well-formed formulas, tautologies, equivalence, normal forms, theory of inference.

#### Combinatorics

Permutation and combinations, partitions, pigeonhole principle, inclusion-exclusion principle, generating functions, recurrence relations.

#### Graph Theory

Graphs and digraphs, Eulerian cycle and Hamiltonian cycle, adjacency and incidence matrices, vertex colouring, planarity, trees.

#### Reference Books

- a) *N. Deo, Graph Theory, Prentice Hall of India, 1974.*
- b) *J.P. Tremblay and R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, Tata McGraw Hill, New Delhi, 2001.*
- c) *C. L. Liu, Elements of Discrete Mathematics, 2nd Edn., Tata McGraw-Hill, 2000.*
- d) *K. H. Rosen, Discrete Mathematics & its Applications, 6th Edn., Tata McGraw-Hill, 2007.*
- e) *V. K. Balakrishnan, Introductory Discrete Mathematics, Dover, 1996.*
- f) *J. L. Hein, Discrete Structures, Logic, and Computability, 3rd Edn., Jones and Bartlett, 2010.*

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## M. Sc. 1<sup>st</sup> Semester

[Compulsory]

### Linear Algebra

#### Review of Vector Spaces

Vector space, subspace, span, linear independence, basis, dimension. Linear transformation: Linear transformation, rank-nullity, Isomorphisms, matrix representation, change of basis, space of linear transformations, Existence of eigenvalues, characteristic polynomial, eigenspace, generalized eigenvectors, Linear functional, Representation of linear transformations by matrices, Dual spaces, Second dual, Reflexive spaces, Invariant subspaces, Direct-sum decomposition, Cyclic subspaces and Annihilators, minimal polynomial, The rational and Jordan canonical forms.

## Inner Product Spaces

Inner products, norm and angle, orthogonality, Gram-Schmidt process, orthogonal complement, best approximation and least squares, Riesz representation and adjoint. Block diagonalization: Diagonalizability, Schurtriangularization, Jordan form.

## Spectral Representation

Operators on inner product spaces, projections, normal operators, self-adjoint operators, singular value decomposition, polar decomposition.

## Reference Books

- a) Axler, S., *Linear Algebra Done Right, Springer International Student Edition (1997)*.
- b) Hoffman, K., Kunze, R.: *Linear Algebra, Prentice Hall Int. Inc., Englewood Cliffs (1971)*.
- c) *Linear Algebra, M.T.Nair and A.Singh, Springer, 2018*.

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**M. Sc. 1<sup>st</sup> Semester**

**[Compulsory]**

**Real Analysis**

## Point Set Topology

Limit points of a set and isolated point of a set, closed sets, dense sets, countable and uncountable sets. Neighbourhood of a point, Interior point, Open set. Union, intersection of open sets and closed sets. Bolzano-Weierstrass theorem, Series and Sequences, Abel's test, Dirichlet's test, Mertens' theorem, Abel's theorem, limit of a function.

## Metrics and Norms

Metric spaces, normed vector spaces, convergence in metric spaces, continuous functions, completeness, Baire's Category Theorem, Contraction mapping theorem, connectedness, Intermediate Value Theorem, Compactness, Heine-Borel Theorem.

## Differentiation

Differentiation, Taylor's theorem.

## Integration

Function of bounded variation, Riemann- Stieltjes integral and relation with Riemann integral, Darboux-Stieltjes integral. Mean value theorems.

## Sequences and Series of Functions

Sequences and series of functions, Uniform convergence, power series and Fourier series, Fourier Series, Dirichlet's Kernel, Riemann- Lebesgue theorem, pointwise convergence of Fourier Series of functions of bounded variation. Weierstrass approximation theorem, Equicontinuity, Arzela-Ascoli theorem.

## Reference Books

- a) *K.R. Stromberg, Introduction to Classical Real Analysis, Wadsworth International, 2017.*
- b) *J.M. Howie, A First Course in Real Analysis, Springer, 2006.*
- c) *H.L. Royden, Real Analysis, Paperback, 1988.*
- d) *R. Robert, G. Bartle, Donald R. Sherbert, Introduction to Real Analysis, 2014.*
- e) *S.C. Malik, Principles of Real Analysis, New Age International Publishers, 2011.*
- f) *S.K. Mapa, Introduction to Real Analysis, Sarat Impressions Pvt. Ltd, 2019.*
- g) *Shanti Narayan, M D Raisinghania, Elements of Real Anyalsis, S. Chand Group, 2008.*

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## M. Sc. 1<sup>st</sup> Semester

[Compulsory]

### Abstract Algebra

#### Group Theory

Review of basic Group Theory, Group Actions, Kernel and Stabilizer of Group Actions, Transitive Group Action, Cayley's Theorem, The Class Equation, Sylow's Theorems, Solvable groups, Nilpotent groups, Direct Products, Structure Theorem for Finite Abelian Groups, Existence and universal Properties of free Groups, Examples of Groups specified by Generators and Relations.

#### Ring Theory

Review of basic Ring Theory, Properties of Ideals, Prime and Maximal Ideals, Two-sided ideals and Quotient Rings, Chinese Remainder Theorem, Euclidean Domain, Euclidean Algorithm, Principal Ideal Domain, Euclidean Domain is a Principal Ideal Domain, UFD, PID implies UFD, Universal Property of a Polynomial Ring, Criteria for Irreducibility, Definition and simple examples of modules over commutative and non-commutative rings.

#### Field Theory

Finite and Algebraic Extensions, Existence and Cardinality of Algebraic Closure, Finite Fields, Galois Theory of Polynomial in characteristic zero and simple examples Classical Straightedge and Compass construction and examples, normal, separable and Galois extensions.

## Reference Books

- a) *D. S. Dummit and R. M. Foote, Abstract Algebra, 2nd Edition, John-Wiley, 1999.*
- b) *S. Lang, Algebra, 3rd Edition, Addison-Wesley, 1999.*
- c) *J.A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.*
- d) *M. Artin, Algebra, Prentice Hall inc 1994.*
- e) *I.N. Herstein, Topics in Algebra, John-Wiley, 1995.*
- f) *T. A. Hungerford, Algebra, Graduate Texts in Mathematics, Springer-Verlag, 1980.*

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**Computer Programming in C++, Matlab/Mathematica (Theory & Lab)**

**Basic Features**

The Matlab workspace about variables, complex number, floating point arithmetic, Mathematical functions.

**Script M files**

Use, Block comments and code cells, startup and finish. Array and Array operations: Simple array, Array addressing or indexing, Array construction, Array orientation, Scalar Array Mathematics, array Manipulation, Array sorting, Sub array searching, Array size, Array and Memory utilization, Multidimensional Array construction & its manipulation. Numeric data type: Integer data type, floating point data types.

**Cell Arrays and Structures**

Cell array creation, its manipulation, Retrieving cell array content, comma separated list, cell functions, cell array of strings, structure creation, structure manipulation, structure functions. Character string: String construction, string evaluation, string functions, cell array of strings.

**Relational and Logical Operations**

Relational and logical operators, Relational and logical functions, Nans and empty, operator precedence. Control flow: For loops, while loops, if else end construction, switch case construction, Try catch blocks.

**Application to Numerical Analysis**

The following topics may be covered in MATLAB. Bisection Method, Newton Raphson Method, Regula Falsi method, Iteration Method, Graffe's Root squaring method, Power Method, L.U. Decomposition Method, Romberg Method, Muller Method, Adams Moulton Method, Newton's Method (forward & backward), Lagrange's interpolation, Divided Difference method, Numerical Differentiation (1st & 2nd Order), Numerical Integration, Least square method, Gauss Elimination method, Gauss Seidal Method, Jacob iteration method, Milne's Method, RungeKutta Method.

**Reference Books**

- a) A. Gilat, *Matlab an Introduction with Applications*, Wiley Publication, 2010.
- b) S. C. Chapra, *Applied Numerical Methods with Matlab for Engineers and Scientists*, Tata McGraw-Hill, 2008.
- c) R. Pratab, *Getting started with Matlab*, 2016.

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## M. Sc. 2<sup>nd</sup> Semester

[Compulsory]

### Topology

Topology on the real line and plane, Topological Spaces, Subspace topology, Bases and Sub-bases, Continuous Functions and homeomorphisms, Connected spaces, Components and Local Connectedness, Path connectedness, Compact spaces, Local compactness. Countabilities and Separation axioms. Product Topology, Quotient Topology. The Urysohn Lemma, The Urysohn Metrization Theorem. The Tietze Extension Theorem, Tychonoff Theorem.

#### Reference Books

- a) J.V. Deshpande, *Introduction to Topology*, Tata McGraw-Hill, 1988.
- b) J. Dugundji, *Topology*, Allyn and Bacon, Inc, 1966.
- c) J.L. Kelley, *General Topology*, Van Nostrand, Princeton, 2014.
- d) M.G. Murdeswar, *General Topology*, New Age International, 2020.
- e) G.F. Simmons, *Introduction to Topology and modern Analysis International Student edition*, 1983.
- f) S. Willard, *General Topology*, Addison Wesley, Reading Mass, 1970.
- g) Seymour Lipschutz, *General Topology*, Mc Graw-Hill, 1965.
- h) K.D. Joshi, *Introduction to General Topology*, New Age International, 1983.
- i) J.R. Munkres, *Topology*, 2nd Ed., Pearson Education India, 2002.

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## M. Sc. 2<sup>nd</sup> Semester

[Compulsory]

### Ordinary Differential Equation

#### Existence and Uniqueness of Solutions

Existence and uniqueness of solution, Picards theorems, Gronwall's inequality, Dependence of solution on initial conditions and on function, Continuation of solutions, Nonlocal existence of solution.

#### Strum-Liouville's System and Green's Function

Strum-Liouville's system, Green's function and its applications to boundary value problems, some oscillation theorems such as Strum theorem, Strum comparison theorem and related results.

#### Homogeneous and Non-homogeneous Equation

System of first order equation, Existence and Uniqueness of solution for systems, fundamental matrix, Non homogeneous linear system, Linear system's with constant as well as periodic coefficients, General theory of homogeneous and non homogeneous linear ODE, Second order linear equations with variable coefficients. Reduction of order when one solution of the homogeneous part is known. Complete solution. Reduction to normal form, Change of independent variable, Transformation of the equation by changing the dependent variables / the independent variable. Wronskian and its properties, variation of parameters, Cauchy-Euler equations.

### Series Solution

Series solutions, Legendre differential equation and Legendre polynomials, Bessel's differential equation and Bessel's function. Laguerre differential equation and Laguerre polynomial, Hermite differential equation and Hermite polynomial; recurrence relations, orthogonal properties.

### Reference Books

- a) *E.A. Coddington: An introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1991.*
- b) *P. Haitman: Ordinary Differential Equations, Wiley, New York, 1964.*
- c) *E.A. Coddington and H. Davinson: Theory of Ordinary Differential Equations, McGraw Hill, NY, 1955.*
- d) *S.C. Deo, Y. Lakshminathan and V. Raghavendra: Text Book of Ordinary Differential Equation (Second Edition) Tata McGraw Hill, New Delhi. 1997*

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## M. Sc. 2<sup>nd</sup> Semester

[Compulsory]

### Complex Analysis

Limits, Continuity and Differentiability, Analytic functions, Harmonic functions and multi-valued functions.

Convergence of numerical series, Radius of convergence of power series, and power series as an analytic function, Laurent series.

Classification of singularities, Cauchy's Residue theorem and evaluation of Real Integrals. Cauchy's integral theorem, Cauchy integral formula, Morera's theorem, Taylor's theorem, Laurent's theorem, Liouville's theorem, Schwarz lemma; Maximum Modulus Principle, Branch of Lagarithm: Branch cut, Branch point.

Conformal mappings, Topology of the complex plane, Stereographic Projections, Riemann sphere, Linear fractional Transformations, Critical Points and Inverse Mappings, Cross ratio, Bilinear Transformation. Exponential and Trigonometric Transformation.

### Reference Books

- a) *Lars V. Ahlfors, Complex Analysis, McGraw-Hill Book Company, Inc., New York, 1986.*
- b) *I. S. Ponnusamy and H. Silverman, Complex Variables with Applications, Birkhauser, Boston, 2006.*
- c) *J.W. Brown and R.V. Churchill, Complex Variables and Applications, McGraw Hill, 2008.*
- d) *H. A. Priestly, Introduction to complex analysis, Clarendon Press, Oxford, 1990.*
- e) *A. I. Markushivich, Theory of Functions of Complex Variables, Vol-I, II, Prentice-Hall, 1965.*
- f) *John B. Conway, Functions of One Complex Variable, Second Edition, Springer International Student-Edition, Narosa Publishing House, 1980.*

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**Statistical Inference and Stochastic Processes**

**Statistical Inference**

**Introduction**

Parametric models, Parameters, Random sample and its likelihood, Statistic and its sampling distributions, Problems of inference. Examples from standard discrete and continuous models such as Bernoulli, Binomial, Poisson, Negative Binomial, Normal, Exponential, Gamma, Weibull, Pareto etc.

**Sufficiency and Estimators**

Concept of sufficiency, minimal sufficiency, Neyman factorization criterion, Fisher information, Exponential families. Maximum likelihood estimators, Method of moment estimators, Percentile estimators, Least squares estimators, Minimum mean squares estimators, Uniformly minimum variance unbiased estimators.

**Statistical Hypothesis**

Simple and composite hypotheses, Null hypotheses, alternative hypotheses, one-sided and two-sided hypotheses. The critical region and test statistic, Type I error and Type II error, Level of significance. Power function of a test, most powerful test. The p-value (observed level of significance), Calculating p-values. Simple hypothesis versus simple alternative: Neyman-Pearson lemma (Statement only).

**Stochastic Process**

**Stochastic Processes**

Definition and examples of stochastic processes, Classifications of stochastic processes.

**Markov Chains**

Definition and examples, Transition Probability matrices, Classification of states of a Markov chain, Determination of higher order transition probabilities, Graph theoretic approach, Reducible Markov chains, Markov Chains with continuous state spaces, Non-homogeneous Markov Chains, Markov chains in continuous time: General pure birth and death processes, Birth and death processes with absorbing states.

**Poisson Processes**

Markov Processes with Discrete State Space: Poisson Process and Its Extensions, Poisson Process, Postulates for Poisson Process, Properties of Poisson Process, Poisson Process and Related Distributions, Inter arrival Time, Further Interesting Properties of Poisson Process, Generalisations of Poisson Process.

**Stochastic Processes in Queuing and Reliability**

General concepts of queuing systems, Steady state and transient behaviour, Birth and death process in queuing theory, Network of Markovian queuing systems, Reliability.

**Introduction to Brownian Motion**

Wiener processes, Differential equations for a Wiener process, Kalmogrov's equations.

## Reference Books

- a) J. O. Berger, *Statistical Decision Theory*, 1980.
- b) E. L. Lehmann, *Testing of Statistical Hypotheses*, 1986.
- c) P. J. Bickel and K. A. Doksum, *Mathematical Statistics*, 2006.
- d) J Medhi, *Stochastic Processes*, New Age International Publishers, 2009.
- e) S.C. Gupta and V. K. Kapoor, *Fundamentals of mathematical Statistics*, Sultan Chand & Sons publication, 2002.
- f) R. L. Berger and G. Casella, *Statistical Inference*, 2006.

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## M. Sc. 2<sup>nd</sup> Semester [Compulsory]

### Numerical Methods and Scientific Computing

#### Numerical Methods

##### Errors in Numerical Computation

Sources of errors and estimations, Propagation of errors, backward error analysis, Sensitivity and conditioning, stability, accuracy, precision floating-point arithmetic.

##### Numerical Solution of Non-linear Equations

Bisection method. Fixed point iteration method, Regula-Falsi and Newton-Raphson methods.

##### Interpolation

Newton's forward and backward interpolation formulae, Stirling's and Bessel's interpolation formulae, Lagrange's and Divided difference interpolation formula, Inverse interpolation, Numerical Differentiation.

##### Numerical Integration

Newton-Cote's formula, Trapezoidal, Simpson's  $\frac{1}{3}$ <sup>rd</sup>, Simpson's  $\frac{3}{8}$ <sup>th</sup> and Weddle's formulae.

##### Numerical Solution of a System of Linear Equations

Gauss elimination method, Gauss Jacobi and Gauss-Seidal method.

##### Numerical Solution or Ordinary Differential Equation

Taylor series method, Picards Method, Euler and modified Euler methods.

#### Scientific Computing

##### Solution of System of Linear and Nonlinear Equations

Triangular factorization, relaxation method. Roots of Polynomial equations: Sensitivity of Polynomial Roots, Steffenson's method, Bairstow's method of quadratic factors, Graffe's root squaring method.

##### Matrices and Eigen Value Problem

LU decomposition of matrices, Power method of extreme eigen values, Jacobi's method for symmetric matrices.

## Interpolation

Hermite interpolation, Spline interpolation – cubic splines, least square approximation to discrete data.

## Integration

Gaussian Legendre and GaussainChebyshevs quadrature, Richardson’s extrapolation, Euler Maclaurin’s sum formula, Romberg integration.

## Differential Equations

First order differential equation: existence, uniqueness, stability of solution,Euler’s method, Multistep predictor corrector method, RungeKutta method.

## Reference Books

- a) *James L. Buchanan and Peter R. Turner: Numerical Methods and Analysis (McGraw-Hill), 1992.*
- b) *John H. Mathews: Numerical Methods for Mathematics, Science and Engineering (Prentice-Hall) , 1992.*
- c) *Kendall E. Atkinson: An Introduction to Numerical Analysis (John Wiley and Sons), 2014.*
- d) *James L. Buchanan and Peter R. Turner: Numerical Methods and Analysis (McGraw-Hill), 1992.*
- e) *M. K. Jain, S. R. K. Iyengar, R. K. Jain : Numerical Methods for Scientific and Engineering Competition (Wiley Eastern), 1985.*
- f) *S.S. Sastry, Introductory methods of numerical analysis, (PHI), 2012.*
- g) *John H. Mathews: Numerical Methods for Mathematics, Science and Engineering (Prentice-Hall) 1992.*
- h) *Kendall E. Atkinson: An Introduction to Numerical Analysis (John Wiley and Sons), 2004.*
- i) *M. K. Jain, S. R. K. Iyengar, R. K. Jain: Numerical Methods for Scientific and Engineering Competition (Wiley Eastern), 1985.*
- j) *James L. Buchanan and Peter R. Turner: Numerical Methods and Analysis (McGraw-Hill), 1992.*

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**M. Sc. 2<sup>nd</sup>Semester**

**[Compulsory]**

**Numerical Methods Lab**

## Experiments to be Handled Using Computer

### NumericalSolution of Algebraic and Transcendental Equations

Write a C program for

(A) Bisection Method

(B) Method of false position and secant method

(C) Newton-Raphson method

### **Numerical Integration**

Write a C program for

- (A) Simpson's one-third rule
- (B) Simpson's three-eighth rule
- (C) Trapezoidal rule of integration

### **Numerical Solution of Ordinary Differential Equations**

Write a C program for

- (A) Euler's method
- (B) Improved Euler's method
- (C) Runge-Kutta second and fourth order methods
- (D) Predictor-corrector methods
- (E) Taylor series method
- (F) Finite Element Problem
- (G) Finite Difference Problem

### **Numerical Differentiation**

Write a C program for

- (A) Numerical differentiation

### **Interpolation Methods**

Write a C program for

- (A) Everette's formula
- (B) Newton's forward and backward interpolation
- (C) Lagrange's interpolation

### **Numerical Solution of System of Linear Equations**

Write a C program for

- (A) Eigen values and eigen vectors
- (B) Method of successive approximation
- (C) Gaussian elimination method
- (D) Gauss-Seidel iterative method
- (E) Inversion of a matrix

### **Reference Books**

- a) Anju Khandelwal, *Numerical Methods And Computer Programming*, Narosa, 2015.
- b) C. Xavier, *C Language and Numerical Methods*, New Age Int. Publishers, 2007.

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## **M. Sc. 3<sup>rd</sup> Semester**

**[Compulsory]**

### **Mathematical Methods**

#### **Integral Transform**

##### **Legendre Transforms**

Introduction, definition of the Legendre Transform and Examples, Basic Operational Properties of Legendre Transforms, Applications of Legendre Transforms to Boundary Value Problems, Applications.

##### **Hermite Transforms**

Introduction, Definition of the Hermite Transform and Examples, Basic Operational Properties, Applications.

##### **Radon Transform**

Introduction, properties of the Radon Transform, Radon Transform of Derivatives and integral, Convolution theorem for the Radon Transform. inverse of the Radon Transform and the Parseval's Relation, Applications.

#### **Integral Equations**

Definition, different types of integral equations, kernels, Conversion of ordinary differential equations into integral equations, eigen value and eigen function problem. Solution of integral equation, Fredholm integral equation of the second kind with separable kernels.

Solution of Fredholm and Volterra integral equations by successive approximation.

Classical Fredholm theory: Statements of Fredholm's first, second and third fundamental theorems and their applications.

Integral equations with symmetric kernels, Hilbert Schmidt theorem and some immediate consequences. Singular integral equations, Integral transform method of solution of integral equations.

Approach to reduce BVP of a self-adjoint DE with homogeneous boundary conditions to integral equation forms.

Auxiliary problem with more general and inhomogeneous boundary conditions. Modified Green's function.

#### **Calculus of Variation**

##### **Variational Problems with Fixed Boundaries**

The variation of a functional and its properties. Euler's equation, functionals dependent on several dependent variables, on higher order derivatives, several independent variables etc. Isoperimetric problems.

##### **Variational Problems with Moving Boundaries**

Transversality conditions, problems related to moving boundaries, one sided variation.

##### **Sufficient Conditions for an Extremum**

Jacobi condition, Legendre condition, weak and strong extremum. Hamilton's canonical equation of motion.



## Variational Methods for Boundary Value Problem

Rayleigh Ritz method, Galerkin's method for BVP in ordinary differential and partial differential equations.

### Reference Books

- a) I.N. Sneddon, *The Use of Integral Transforms*, Tata McGraw Hill, 1979.
- b) B.P. Parasar, *Differential equation and integral equation*, CBS Publications, 2008.
- c) Petrovsky, *Lectures on the Theory of Integral Equations*, Mir Publication, 1971.
- d) G. Yankovsky, *Problems and exercise in Integral Equation*, Mir Publication, 1971.
- e) R.R. Goldberg, *Fourier Transform*, Cambridge Univ. Press, 2009.
- f) Gelfand, I. M. and Fomin, S. V., *Calculus of Variations*, revised English edition translated and edited by Richard A. Silverman, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963.
- g) Krasnov, M. L.; Makarenko, G. I. and Kisel'ev, A. I., *Problems and Exercises in Integral Equations*, translated from the Russian by George Yankovsky, 1975.
- h) Lokenath Debnath, Dambaru Bhatta, *Integral transforms and their applications*, Taylor & Francis, 2006.
- i) Brunt, Bruce van, *The Calculus of Variations*, Springer, 2004.
- j) Pipkin, Allen C., *A Course on Integral Equations*, Texts in Applied Mathematics, 9, Springer-Verlag, New York, 1991.

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**M. Sc. 3<sup>rd</sup> Semester**

**[Compulsory]**

**Partial Differential Equation**

### Higher Order Partial Differential Equation

Homogeneous equations with constant coefficients, Cauchy Problems for first Order Hyperbolic Equations, Classification of second order partial differential equation, Monge's Method.

### Heat Equation

Solution by the method of separation of variables, Applications to one dimensional heat flow, Properties of solutions.

### Wave Equation

Solution by the method of separation of variables, Solution by spherical means and Riemann method of solution, Applications to vibration of strings.

### Laplace's Equation

Solution by the method of separation of variables, Boundary value problems, maximum and minimum principles, uniqueness and continuity theorems, Dirichlet problem for a circle, Dirichlet problem for a circular annulus, Neumann problem for a circle, theory of Green's function for Laplace's equation.

### Reference Books

- a) I.N. Sneddon, *Elements of Partial Differential Equations*, McGraw Hill Book Company, 1957.
- b) Phoolan Prasad, Renuka Ravindran, *Partial Differential Equations*, Wiley Eastern Limited, 1987.

- c) *F. John, Partial Differential equations, Springer,1982.*
- d) *T. Amarnath, An Elementary Course in Partial Differential Equations,2003.*
- e) *L.C. Ivans, M Graduate,Partial Differential Equations Studies in Mathematics, Volume 19, AMS, 1968.*

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## M. Sc. 3<sup>rd</sup> Semester

**[Compulsory]**

### Measure Theory

#### The Lebesgue Measure

Outer measure, Measurable sets and Lebesgue measure, Properties of Measurable sets,  $\sigma$ -Algebras, Borel sets and their measurability, Non measurable sets.

#### Measurable Functions

Definition and properties of Measurable functions, Sets of Measure Zero, Borel measurable functions, Sequence of functions, Littlewood's three principles, Convergence in measure.

#### Lebesgue Integral

The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, The integral of a nonnegative function, The general Lebesgue integral.

#### Differentiation and Integration

Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity, Convex Functions.

#### The Lebesgue $L^p$ Spaces

The  $L^p$  spaces, The Minkowski and Holder inequalities, Convergence and completeness, Approximation in  $L^p$ . Bounded linear functionals on the  $L^p$  spaces.

#### Reference Books

- a) *I.K. Rana, An Introduction to Measure and Integration, Second Edition, Narosa, 2005.*
- b) *G. De. Barra, Measure Theory and Integration, Horwood Publishing Corporation, 2003.*
- c) *P.K. Jain, V.P. Gupta, Lebesgue Measure and Integration, New Age International, 2006.*
- d) *H. L. Royden, Real Analysis (Third Edition), Macmillan Publishing Company, 1988.*

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## M. Sc. 3<sup>rd</sup> Semester

**[Elective]**

### Data Structure and Algorithm

#### Basic Concepts

Mathematical Background; Complexity Analysis; Arrays: one dimensional, multi-dimensional, Sparse Matrix, Elementary Operations; Asymptotic notations-  $o$ ,  $O$ ,  $\theta$ ,  $\Theta$ ,  $\omega$ ,  $\Omega$  and their properties.

## Preliminaries

Growth of functions, recurrence relation, generating functions, solution of difference equations, Master's theorem (without proof). Sorting and Order Statistics: Bubblesort, mergesort, heapsort, quicksort, sorting in linear time, median and order statistics.

## Linear Data Structure

Stacks: Representation, elementary operations and applications such as infix to postfix, postfix evaluation, parenthesis matching; Queues: Simple queue, circular queue, dequeue, elementary operations and applications; Linked lists: Linear, circular and doubly linked lists, elementary operations and applications such as polynomial manipulation.

## Non - Linear Data Structure

Trees: Binary tree representation, tree traversal, complete binary tree, heap, binary search tree, height balanced trees like AVL tree and 2-3 tree, tries, red-black tree, B-tree, other operations and applications of trees; Graphs: representation, Adjacency list, graph traversal, path matrix, connected components, DAG, topological sort, Spanning tree; Sorting: Selection sort, bubble sort, quick sort, merge sort, heap sort, radix sort; Searching: linear and binary search; Hashing: hash tables, hash functions, open addressing.

## File Structures

Introduction, data file types, file organization, file access methods.

## Algorithms Design Techniques

Divide-and-conquer method. Greedy method. Dynamics programming technique. Branch-and-bound method. Examples. Representation of graph in a computer. Binary tree traversals. Spanning tree. Shortest path problem. NP-Completeness.

## Reference Books

- a) *Alfred V. Aho, Jeffrey D. Ullman, John E. Hopcroft, Data Structures and Algorithms, Addison Wesley, 1983.*
- b) *M.A. Weiss, Data Structures and Algorithm Analysis in C++, 3rd Edn, Pearson, Addison Wesley, 2006.*
- c) *A.M. Tenenbaum, Y. Langsam, and M.J. Augenstein, Data Structures using C, PHI, New Delhi, 2009.*
- d) *S. K. Srivastava, Data Structures Through C in Depth, BPB Publications, 2011.*
- e) *T.H. Cormen, C.E. Leiserson, R.L. Rivest, and C. Stein, Introduction to Algorithms, 3rd Edn, PHI, New Delhi, 2009.*
- f) *Debasis Samanta, Classical Data Structure, PHI Pvt. Limited. 2011.*

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**M. Sc. 3<sup>rd</sup> Semester**

**[Elective]**

**Operation Research – I**

## Goal Programming

Introduction, Difference between LP & GP approach, Concept of Goal Programming, Graphical solution-method of GP, Modified simplex method of GP.

## **Dynamic Programming**

Introduction, Nature of Dynamic Programming, Deterministic processes, Non- sequential discrete optimization, Allocation problems, Assortment Problems, Sequential discrete optimization, Long-term planning problem, Multi-stage decision process, Application of Dynamic Programming in production scheduling and routine problems.

## **Inventory Control**

Inventory control – Deterministic including price breaks and Multi-item with constraints. Probabilistic (with and without lead time). Fuzzy and Dynamic inventory models.

## **Queuing Theory**

Basic Structure of queuing models, Poisson queues, M/M/I, M/M/C or finite and infinite queue length, Non-Poisson queue- M/G/I, Machine-Maintenance (steady state).

## **Network: PERT and CPM**

Introduction, Basic difference between PERT and CPM. Steps of PERT/CPM Techniques, PERT/ CPM Network Components and precedence relationships, Critical path analysis, Probability in PERT analysis, Project Time-Cost, Trade-Off. Updating of the project, Resource allocation – resource smoothing and resource levelling.

## **Replacement and Maintenance Models**

Introduction. Failure Mechanism of items, Replacement of items deteriorates with time, Replacement policy for equipment when value of money changes with constant rate during the period. Replacement of items that fail completely individual replacement policy and group replacement policy. Other replacement problems staffing problem. Equipment renewal problem.

## **Simulation**

Introduction. Steps of simulation process, Advantages and disadvantages of simulation. Stochastic simulation and random numbers. Monte – Carlo simulation. Random number. Generation. Simulation of inventory problems, Simulation of Queuing problems, Role of computers in Simulation. Application of Simulations.

## **Reference Books**

- a) *A.P. Verma, Introduction to Operations Research, S.K. Kataria & Sons, 3rd Edition 2014.*
- b) *Hamdy A. Taha, Operations Research: An Introduction, 10th Edition, Pearson Education India, 2017.*
- c) *P K Gupta, Operations Research, S Chand, Fifth Edition, 1976.*
- d) *J K Sharma, Operations Research, Theory and Applications, Macmillan India Ltd., 1997.*
- e) *D.S Hira and P K Gupta, Operations Research, S Chand, 2015.*

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**Differential Geometry**

**Differentiable Manifold**

Calculus on  $\mathbb{R}^n$ : Continuity and differentiability of function from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ , Inverse function theorem, Implicit function theorem, the existence and uniqueness theorem of solution of ODE. Smooth manifold, Differentiable manifold, Tangent space, Derivative of a smooth map, Tangent bundle, Immersion, submersion, embedding, sub-manifold; regular and critical point, Whitney weak embedding theorem and its consequences.

**Vector Fields**

Height of the level set, level curves, Lie bracket, Lie algebra, Lie derivative, integral curve of a vector field, flows and local flows, existence of integral curve, complete vector field, existence of complete vector field, vector fields related by a differentiable map. Distribution: Involutive distribution, the Frobenius theorem and its applications.

**Surfaces**

Hyperplane, Lagrange multiplier, Vector Fields on Surfaces, maximal integral curve, orientation and its consistency, Osculating plane, Serret-Frenet formula, first and second fundamental form, the Gauss map spherical image.

**Geodesics**

Maximal geodesic, covariant derivative and acceleration, Fermi derivative, The Weingarten Map geodesic flow.

**Curvature of Plane Curves**

Centre of curvature, radius of curvature, Isometries, Intrinsic differentiation, Gauss-Kronecker curvature, Fundamental theorem on curves. Curvature of surfaces: Parametrized surfaces, local equivalence of surfaces. Gauss-Bonnet Theorem and its application to constant curvatures, theorems of Hadmard.

**Lie Group and Lie Algebra**

Left invariant vector fields, exponential map and its applications, Lie algebra homomorphism, One parameter subgroups, Adjoint representation.

**Reference Books**

- a) J. A. Thorpe, *Elementary Topics in Differential Geometry* (Springer), 2004.
- b) Manfredo. P. DoCarmo, *Differential Geometry of Curves and Surfaces*, Prentice Hall, 1976.
- c) J.J. Stoker, *Differential Geometry*, Wiley-Interscience, 1969.
- d) Andrew Pressley, *Elementary Differential Geometry*, Springer – 2002.
- e) B. O'Neill, *Elementary Differential Geometry*, Academic Press, 1966.

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**Number Theory and Cryptography**

**Number Theory**

**Basis Representation**

Principles of mathematical induction, The basis representation theorem, The fundamental theorem of arithmetic: Euclid's division lemma, Divisibility, The linear Diophantine equation, The fundamental theorem of arithmetic.

**Combinatorial and Computational Number Theory**

Fermat's little theorem, Wilson's theorem, Generating functions, The use of computers in number theory, Fundamentals of congruences, Basic properties of congruences, Residue systems, Riffing.

**Solving Congruences**

Linear congruences, The Theorems of Fermat and Wilson revisited, The Chinese remainder theorem, Polynomial congruences.

**Arithmetic Functions**

Combinatorial study of  $\varphi(n)$ , Formulae for  $d(n)$  and  $\varphi(n)$ , Multiplicative arithmetic functions, The Mobius inversion formula, Primitive roots: Properties of reduced residue systems, Primitive root modulo  $p$ .

**Cryptography**

**Introduction**

Basic objects of Cryptography, secret-key and public-key cryptography, one-way and trapdoor one-way functions, Cryptanalysis attack models, Classical cryptography.

**Block & Stream Ciphers**

Modes of operation, DES and its variants, RCS, IDEA, SAFER, FEAL, Blowfish, AES, linear and differential cryptanalysis Stream ciphers based on linear feedback shift registers, SEAL, Unconditional security.

**Message Digest**

Properties of Hash function, MD2, MD5 and SHA-1, keyed hash function, attack on hash function.

**Public Key Parameters**

Modular arithmetic, GCD, Primality testing, Modular square root, finite fields.

**Intractable Problems**

Integer factorisation problem, RSA problem, Modular square root problem, discrete logarithm problem, Diffie-Hellman problem, known algorithm for solving the Intractable Problems.

**Public key Encryption**

RSA, Rabin and ElGamal schemes, side channel attacks, Key Exchange: Diffie-Hellman and MQV algorithms.

## Reference Books

- a) Douglas R. Stinson, *Cryptography, Theory & Practice, Second Edition*, CRC Press, 2002.
- b) Alfred J. Menezes, Paul C. Van Oorschot, Scott A. Vanstone, *Handbook of Applied Cryptography*, CRC Press, 2001.
- c) Johannes. A. Buchmann, *Introduction to Cryptography*, Springer, September 2000.
- d) Steven D. Galbraith, *Mathematics of Public Key Cryptography*, Cambridge university press, 2012.
- e) Jonathan Katz, Yehuda Lindell, *Introduction to Modern Cryptography*, Chapman & Hall/CRC, 2007.
- f) Jay R Goldman, *The Queen of Mathematics, a historically motivated guide to number theory*, A K Peters Ltd, 2017.
- g) SabanAlaca, Kenneth S Williams, *Introduction to Algebraic Number Theory*, Cambridge University Press, 2004.
- h) Richard A Mollin, *Advanced Number Theory with Applications*, CRC Press, A Chapman & Hall Book, 2017.
- i) Kenneth. H. Rosen, *Elementary Number Theory & Its Applications*, AT&T Bell Laboratories, Addition Wesley Publishing Company, 1993.
- j) Kenneth Ireland & Michael Rosen, *A Classical Introduction to Modern Number Theory*, 2nd edition, Springer-verlag, 1990.

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## M. Sc. 3<sup>rd</sup> Semester

[Elective]

### Soft Computing – I

#### Introduction

Introduction to Soft Computing, Components of Soft Computing, Importance of Soft Computing.

#### Fuzzy Set Theory

Fuzzy sets, Characteristics function and definition of fuzzy sets,  $\alpha$ - level Fuzzy sets: Characteristics function and definition of fuzzy sets, fuzzy point, sets, convex fuzzy sets, basic operations on fuzzy sets. Cartesian products, algebraic products, bounded sum and difference, t-norms and t-co norms, quasi-coincidence of two fuzzy subsets. Fuzzy numbers, triangular fuzzy numbers.

#### Fuzzy Optimization

Linear Programming Problems with Fuzzy Recourses:

(i) Verdegay's approach and (ii) Werner's approach. L.P. with fuzzy resources and objectives.

Zimmermann's approach, L.P.P. with fuzzy parameters in the objective function. Fuzzy multi-objective Linear Programming Problem. Methodologies of solving fuzzy M.O.L.P.

#### Evolutionary Algorithm

Genetic Algorithms; Evolution Strategies; Evolutionary Programming; Genetic Programming; Differential Evolution and modified methods. Ant Colony Optimization. Particle Swarm Optimization and some recent Algorithms.

## Reference Books

- a) Jang J.S.R, Sun C.T., Mizutani E. *Neuro, Fuzzy and Soft Computing*, 1996.

- b) *Ama Haykin. An Introduction to Neural Networks, 2018*
- c) *Goldberg. Genetic Algorithms in Search, Optimization and Machine Learning, 1989.*
- d) *Klir and Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, 1995.*
- e) *S. N. Sivanandam, S.N.Deepa, Principles of Soft Computing, Wiley Publication, 2011.*
- f) *A.DasBhattacharjee, Artificial Intelligence and Soft Computing for Beginners, 2014.*
- g) *H.J Zimmermann, Fuzzy Set Theory - and Its Applications, Allied Publisher's Limited, 2007.*
- h) *Robert Babuška, Fuzzy Modelling for Control, International Series in Intelligent Technologies, Volume 12, 1998.*

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## M. Sc. 3<sup>rd</sup> Semester

[Elective]

### Mathematical Modelling and Simulation – I

#### Mathematical Models of Population Biology or Ecology

##### Mathematical Models

Deterministic and Stochastic. Single species population models. P-V Logistic equation. Population growth model– An age structured model.

##### Interactions between Two Species

Host-Parasite type of interactions, Competitive type of interactions. Trajectories of interactions of H-P and competitive types between two species. Effect of migration on H-P interactions. Some consequences of Lotka-Volterra equations. Generalized L-V equations. Constant of motion in the dynamical system.

##### Stochastic Processes and Need of Stochastic Models

Pure birth process, Pure death process, Birth and death process. Linear birth-death-immigration-emigration processes. Effects of both immigration and emigration on the dynamics of population.

##### Biological Mechanisms Responsible for "Time-Delay"

Discrete and continuous time-delay. The single species logistic model with the effect of time-delay. Stability of equilibrium position for the logistic model with general delay function. Stability of logistic model for discrete time lag. Time-delayed H-P model together with their stability analysis.

#### Mathematical Theory of Epidemics

##### Introduction

Some basic definitions. Simple epidemic model, General epidemic model. Kermack-McKendrick threshold theorem. Recurring epidemic model. A comparative study of these models.

##### Control of an Epidemic

Stochastic epidemic model without removal. Models having multiple infections. Epidemic model with multiple infections. Stochastic epidemic model with removal. Stochastic epidemic model with removal, immigration and emigration. Special discussion on the stochastic epidemic model with carriers.



## Simple Extensions of SIR Model

Different case studies - (i) Loss of immunity, (ii) Inclusion of immigration and emigration, (iii) Immunization. SIR endemic disease model.

## Reference Books

- a) X. Q. Zhao, *Dynamical Systems in Population Biology*, Canadian Mathematical Society, (2017).
- b) R. M. Andersson and R M May, *Infectious Diseases of Humans*, (1992).
- c) J. N. Kapur, *Mathematical Models in Biology and Medicine*, East West Press Pvt Ltd (1985).
- d) R. Habermann, *Mathematical Models*, Prentice Hall (1977).
- e) R. W. Poole, *An Introduction to Quantitative Ecology*, McGraw- Hill, (1974).
- f) E. C. Pielou, *An Introduction to Mathematical Ecology*, Wiley, New York (1977).
- g) R. Rosen, *Foundation of Mathematical Biology (vol. I& II)*, Academic Press, (1972).
- h) Mark Kot, *Elements of Mathematical Ecology*, Cambridge University Press (2003)
- i) J. D. Murray, *Mathematical Biology*, Springer-Verlag, Berlin (1989).

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## M. Sc. 4<sup>th</sup> Semester

[Elective]

### Functional Analysis

#### Introduction to Normed Space

Normed linear space, Hölder's inequality, Minkowski's inequality, Banach space, equivalent norms, Riesz lemma, Finite dimensional normed spaces and subspaces, quotient space, Compactness and finite dimension.

#### Finite Dimensional Normed Space

Bounded and continuous linear operators, Linear functional, Linear operators and functional on finite dimensional spaces, Dual of normed spaces, reflexive space, open mapping theorem and closed graph theorems, Uniform boundedness principle.

#### Inner Product Spaces

Inner product spaces, Cauchy- Schwarz inequality. Parallelogram law, Pythagorean, theorem, Bessel's Inequality, Gram-Schmidt orthogonalisation process, Hilbert space Examples, orthonormal sets. Complete orthonormal sets and Parseval's Inequality. Orthogonal complement. Projection theorem, Riesz Representation theorem.

#### Operators on Inner Product Spaces

Adjoint of an operator on Hilbert space, properties of adjoint operation, self-adjoint operator and its characterisation, Projection operator, unitary operator, concept of normal operator and its characterisation, normed operator, unitary operator and its characterisation. Zorn's lemma, Hahn-Banach theorem, Topological vector spaces.

## Reference Books

- a) Bachman and Naric, *Functional Analysis*, Academic Press (1966).

- b) *G. F. Simmon, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company (1963).*
- c) *Goffman and Pedrick, First Course in Functional Analysis, Prentice-Hall, Inc.1983*
- d) *John B. Conway, A Course in Functional Analysis, Springer (1990).*
- e) *A. E. Taylor, Introduction to Functional Analysis, John Wiley & Sons. (1958).*
- f) *B. V. Limaye, Functional Analysis, New Age International Ltd.2014*
- g) *M. Thamban Nair, Functional Analysis, Prentice-Hall of India Pvt. Ltd., New Delhi (2002).*
- h) *Jain, Ahuja and Ahmad, Functional Analysis, New Age International (P) Ltd. (1997).*
- i) *Kreyszig Erwin, Introductory Functional Analysis with Applications, Wiley Classics Library, John Wiley & Sons, Inc., New York (1989).*
- j) *Walter Rudin, Functional Analysis, Tata McGraw-Hill (1974).*

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## M. Sc. 4<sup>th</sup> Semester

[Elective]

### Advanced Numerical Analysis

#### Finite Difference Methods for PDEs

Basics on Finite Differences. Finite Difference Approximation of the Laplacian in Two Dimensions. The Discrete Maximum Principle for a Finite Difference Operator. Stability and Convergence of the Finite Difference Approximation of the Poisson Problem with Dirichlet Boundary Conditions. An Efficient Solver for the Dirichlet Problem in the Rectangle. Approximate solution of Fredholm equation by finite sums and degenerate Kernels. Numerical approximation of Volterra equations.

#### Finite Volume Methods

Different types of finite volume grids, approximation of surface and volume integrals, interpolation methods.

#### Finite Element and Boundary Element Methods

Weighted residual method, Galerkin, Least square, partition, moment and collection methods. Solution of boundary value problems by Ritz method. Finite elements and boundary elements of various terms. Constant elements by Gaussian quadrature. Numerical integration over finite elements. Solution boundary value problems by Finite element and Boundary element methods.

#### Reference Books

- a) *Isacson and Keller, Analysis of Numerical methods, 1994.*
- b) *Ralston and Rabinowitz, A first course in Numerical Analysis, 2012.*
- c) *G.D.Smith, Numerical solution of partial differential equations, 1985.*
- d) *B.P. Demidovich, J.A.Maron, Computational Mathematics.*
- e) *A. Gourdin, M. Boumahrat, Applied Numerical Methods, 2004.*
- f) *M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods for Scientific and Engineering Competition (Wiley Eastern), 1985.*
- g) *A.R.Mitchell, The finite elements method in partial differential equations, 1977.*
- h) *Prem K. Kytbe, An introduction to boundary element methods, 1995*

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**Operation Research – II**

**Stochastic Programming**

Chance constraint programming techniques.

**Geometric Programming**

Geometric programming (both unconstrained and constrained), Fuzzy geometric programming.

**Games**

Preliminaries concepts of continuous Game, Bi-matrix Games, Nash equilibrium, solution of Bi-matrix Games through quadratic programming (relation with non-linear programming)  
Multi-objective Linear & Non-linear programming, Complete optimal solution, Pareto & Weak Pareto optimal solution, Utility function method , Global criterion method, Fuzzy programming technique.

**Optimal Control**

Performance indices, methods of calculus of variation, Transversally conditions, simple optimal problems of Mechanics, Pontryagin's Principle (with proof assuming smooth condition), Linear regulator, application of dynamic programming in proving Pontryagin's Principle, Bang-Bang controls.

**Sequencing**

Problem with n jobs two machines, n jobs three machines and n jobs m machines.

**Reliability**

Concepts, Reliability definition, system Reliability, system failure rate, Reliability of the systems connected in series or/ and parallel.

**Information Theory**

Introduction, communication process-memory less channel, the channel matrix, probability relation in a channel, noiseless channel. Measure of information—Properties of Entropy function, measure of other information quantities—marginal and joint Entropies, conditional Entropies, expected mutual information, axiom for an Entropy function, properties of Entropy function. Channel capacity, Efficiency and Redundancy.

**Encoding**

Objectives of encoding, Shannon-Fano Encoding Procedure, Necessary and sufficient condition for noiseless encoding.

**Reference Books**

- a) *A.P. Verma, Introduction to Operations Research, S.K. Kataria & Sons, 3rd Edition, 2014.*
- b) *Hamdy A. Taha, Operations Research: An Introduction, 10th Edition, Pearson Education India, 2017.*
- c) *P. K. Gupta, Operations Research, S Chand, Fifth Edition, 1976.*
- d) *J. K. Sharma, Operations Research, Theory and Applications, Macmillan India Ltd., 1997.*
- e) *D.S. Hira, P. K. Gupta, Operations Research, S Chand, 2015.*

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## **M. Sc. 4<sup>th</sup> Semester**

**[Elective]**

### **Mechanics**

#### **Classical Mechanics**

##### **Coordinate System and Basic Concepts**

Generalized co-ordinates, holonomic and non holonomic systems, unilateral and bilateral constraints; principle of virtual work, D'Alembert's principle.

##### **Variational Principles**

Variational principles, problems of mechanics, moving problems of calculus of variations, shortest distance, minimum surface of revolution, Brachistochrone problem, iso-perimetric problem, geodesic, fundamental, lemma of calculus of variations.

##### **Lagrange's Equations of Motion**

Lagrange's equations, uniqueness of solution, Energy equation of conservative fields.

##### **Hamilton's Principle**

Generalized momentum, problem of Liouville type, cyclic coordinates. Hamilton's principle, Hamiltonian, principle of least action.

##### **Hamiltonian Dynamics**

Derivation of Lagrange's equations from Hamilton's principle, Routh's equation, Hamilton-canonical equation of motion.

##### **Poisson Bracket**

Poisson bracket, Poisson's identity, Liouville's theorem, Jacobi-Poisson theorem.

##### **Canonical Transformations**

Canonical transformations, Legendre transformations, generating functions, bilinear invariant condition.

#### **Continuum Mechanics**

##### **Stress**

Body force, Surface force, Cauchy's stress principle. Stress vector, State of stress at a point, Stress tensor, The stress vector- stress tensor relationship. Force and moment equilibrium, Stress tensor symmetry, Stress quadric of Cauchy, Stress transformation laws, Principal stress, Stress invariant, Stress ellipsoid.

##### **Strain**

The Continuum hypothesis. Deformation Gradients, Displacement Gradient, Deformation tensor, Finite strain tensors, Small deformation theory—infinitesimal strain tensor, Relative displacement, Linear rotation tensor, Interpretation of the Linear strain tensors, Strain ratio, Finite strain interpretation, principal strains, strain invariant, cubical dilatation, Compatibility equation for linear strain, Strain energy function. Hook's Law. Saint Venant's Principle, Airy's strain function. Isotropic media, Elastic constraints, Moduli of elasticity of isotropic bodies and their relations, displacement equation of motion. Waves in isotropic elastic media. Perfect fluid, Kinematics of fluid,

Lagrangian method, Eulerian method, Acceleration, Equation of continuity, The boundary surface, Stream lines and Path lines, Irrotational motions and its physical interpretation, Velocity potential, Euler's equation of motion of an inviscous fluid, Cauchy integral, Bernouli's equation , Integration of Euler's equation.

## Reference Books

- a) *Rana and Joag, Classical Mechanics, Narosa Publications, 1991.*
- b) *D.N. Berghese and A.M. Downs, Classical mechanics and Control, John Willey.*
- c) *E.T. Whittecker, Treatise on the Analytical Dynamics and Rigid Bodies, 1988.*
- d) *I.S. Sokolnikoff, Mathematical Theory of Elasticity , Tata Mc. Grawhill, 1997.*
- e) *S. Valliappan, Continuum Mechanics , Oxford & IBH Publishing Company, 1981.*
- f) *L.E. Malvern, Introduction to the Mechanics of a continuous medium, Prentice - Hall, Inc, 1977.*
- g) *J. B. Marion, S. T. Thornton, Classical Dynamics of Particles and Systems-, Harcourt Brace Jovanovich, 1988.*
- h) *Scheck, Florian, Mechanics - From Newton's Laws to deterministic Chaos, Graduate Texts in Physics, Springer, 2010.*
- i) *J. E. Marsden, Ratiu, Tudor, Introduction to Mechanics and Symmetry, Texts in Applied Mathematics, Springer-Verlag, 1999.*
- j) *H. Goldstein, Poole. C. P., J. Safko, Classical Mechanics, Pearson, 2000.*
- k) *F. Chorlton, Textbook of Dynamics, Ellis Horwood Series, Mathematics and its Applications, Halsted Press (John Wiley & Sons, Inc.), 1983.*
- l) *H. Goldstein, Poole. C. P. and J. Safko, Classical Mechanics, Pearson, 2002.*
- m) *F. Chorlton, Textbook of Dynamics, Ellis Horwood Series: Mathematics and its Applications, Halsted Press (John Wiley & Sons, Inc.), 1977.*
- n) *J. E. Marsden, Ratiu, Tudor, Introduction to Mechanics and Symmetry, Texts in Applied Mathematics, Springer-Verlag, 1998.*
- o) *S.C. Deo, Y. Lakshminathan and V. Raghavendra, Text Book of Ordinary Differential Equation (Second Edition) Tata McGraw Hill, New Delhi, 1997.*

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**M. Sc. 4<sup>th</sup> Semester**

**[Elective]**

**Fluid Mechanics**

Review of gradient, divergence and curl. Elementary idea of tensors. Velocity of fluid, Streamlines and path lines, Steady and unsteady flows, Velocity potential, Vorticity vector.

Continuum hypothesis, forces acting on a fluid, stress tensor, analysis of relative motion in the neighbourhood of a point, Euler's theorem, Reynolds transport theorem, conservation of mass, material surface, momentum equation.

Stream lines, Bernoulli's theorem, circulation, Kelvin's circulation theorem, vorticity, Lagrange's theorem on permanence of vorticity, two dimensional irrotational flow of an incompressible fluid, Milne-Thomson circle theorem, Blasius theorem, flow past an airfoil, the Joukowski transformation, Theorem of Joukowski and Kutta.

Axisymmetric flows, Stokes stream function, Butler's sphere theorem, flows due to source, doublet, uniform flow past a sphere, irrotational three dimensional flow, Weiss' sphere theorem.

Constitutive equations for incompressible fluids, derivation of Navier-Stokes equations. Simple exact solutions of Navier-Stokes equation: (i) Plane Poiseuille and Hagen-Poiseuille flows (ii) Generalized plane Couette flow (iii) Steady flow between two rotating concentric circular cylinders (iv) Stokes's first and second problems. Dynamical similarity and Reynolds number.

### Reference Books

- a) *G.K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, 1970.*
- b) *F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers & Distributors, 2000.*
- c) *H. Lamb, Hydrodynamics, University Press, 1916.*
- d) *C.S. Yih, Fluid Mechanics, McGraw-Hill Book, Company, 1988*

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## M. Sc. 4<sup>th</sup> Semester

[Elective]

### Soft Computing-II

#### Fuzzy Systems

Arithmetic operations on Fuzzy intervals and Fuzzy numbers, Lattice of Fuzzy numbers, L-R representation of fuzzy numbers, Fuzzy equations.

#### Generalization and Variants of Fuzzy Sets

L-fuzzy sets, interval-valued fuzzy sets, Type 2 fuzzy sets, intuitionistic fuzzy sets and set operation of intuitionistic fuzzy sets, The Zadeh's extension principle.

#### Fuzzy Logic

Related Definitions, Fuzzy Expert Systems, Fuzzy Inference Rules and Fuzzy Reasoning, Introduction to Fuzzy logic, Fuzzy rule generation, Linguistic variables.

#### Hybrid Systems

Neuro-genetic systems, Hybrid Evolutionary Algorithms, Fuzzy Neural Systems.

### Reference Books

- a) *Jang J.S.R, Sun C.T. and Mizutani E. Neuro, Fuzzy and Soft Computing, 1997*
- b) *Ama Haykin, An Introduction to Neural Networks. 2008*
- c) *Klir and Yuan, Fuzzy Sets and Fuzzy Logic Goldberg, Genetic Algorithms, 2015.*

- d) S. N. Sivanandam, S.N.Deepa, *Principles of Soft Computing*, Wiley Publication, 2007.
- e) A. Das Bhattacharjee, *Artificial Intelligence and Soft Computing for Beginners*, 2018.
- f) H. J Zimmermann, *Fuzzy Set Theory - and Its Applications*, Allied Publisher's Limited, 2014.
- g) George Klir, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Pearson, 2015.
- h) Robert Babuška, *Fuzzy Modelling for Control*, International Series in Intelligent Technologies, Volume 12, 2012.

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## M. Sc. 4<sup>th</sup> Semester

[Elective]

### Mathematical Modelling and Simulation – II

#### Some Mathematical Aspects of Oscillations of the Biological Systems

Introduction; Biological Clock; Model for the circadian oscillator. Pharmacokinetics. Mathematical models in Pharmacokinetics -Compartmental Analysis. Technique. Two compartment model - Clinical Bromsulphalein (BSP) Test. Basic equations for an n-compartment system. Distributions of drugs in n- compartment model for (i) given initial dose (ii) repeated medication, (iii) constant rate of infusion and (iv) truncated infusion. Compartment model for diabetes mellitus. Stochastic compartment models. Drug action. Some general principles for real biological oscillations. Cellular mechanism and genesis of Atherosclerosis.

#### Arterial Biomechanics

Importance of studies on the mechanics of blood vessels. Structure and functions of blood vessels; Mechanical properties. Viscoelasticity; Linear discrete viscoelastic (spring-dashpot) models: Maxwell Fluid, Kelvin Solid, Kelvin Chains and Maxwell models. Creep Compliance, Relaxation Modulus. Hereditary Integrals, Stieltjes Integrals.

Constituents of blood. Structure and functions of the constituents of blood. Mechanical properties of blood. Equations of motion applicable to blood flow. Non-Newtonian fluids - Power law, Bingham Plastic, Herschel-Bulkley and Casson fluids. Steady non- Newtonian fluid flow in a rigid circular tube. Fahraeus-Lindqvist effect. Pulsatile flow in both rigid and elastic tubes. Blood flow through arteries with mild stenosis. Shear stress on surface of the stenosis; Two-layered flow in a tube with mild stenosis.

Large deformation theory. Various forms of strain energy functions. The base vectors and metric tensors; Green's deformation and Lagrangian strain tensors. Cylindrical model; Constitutive equations for blood vessels; Equations of motion for the vascular wall.

#### Biological Diffusion and Diffusion-Reaction Models

Fick's laws of diffusion, One-dimensional diffusion model and its solution, Some solutions of two-dimensional diffusion equation, Various modifications of diffusion equation to diffusion-reaction models arising in pharmacokinetics and ecology

Hemodialyser and dialysis of blood, Basic equations for a circular-duct and a parallel-plate dialyser, Pecletnumber, Sherwood number, Solutions of basic equation for a circular-duct dialyser by (i)

separation of variables method and (ii) Galerki's method. Solution for parallel-plate dialyser

### Reference Books

- a) D. A. MacDonald, *Blood Flow in Arteries*, The Williams and Wilkins Company, Baltimore , 1974.
- b) Y. C. Fung, *Biomechanics of Soft Biological Tissues*, Springer Verlag.
- c) R. Habermann, *Mathematical Models* , Prentice Hall, 1977.
- d) R. W. Poole, *An Introduction to Quantitative Ecology*, McGraw- Hill, 1974.
- e) E. C. Pielou, *An Introduction to Mathematical Ecology*, Wiley, New York, 1977.
- f) R. Rosen, *Foundation of Mathematical Biology (vol. I& II)*, Academic Press, 1973.
- g) W Flugge, *Viscoelasticity*, Springer-Verlag, 1975.
- h) M Zamir, E L Ritman, *The Physics of Pulsatile Flow*, 2012.
- i) J. N. Kapur, *Mathematical Models in Biology and Medicine*, East West Press Pvt Ltd, 1985.

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## M. Sc. 4<sup>th</sup> Semester

[Elective]

### Graph Theory

#### Introduction to Graphs

The concept of a graph, Paths in graphs, Graph models, Graph terminology and special types of graphs, Bipartite graphs, Complete graphs, External graphs, Intersection graphs, Operations on graph, Graph Isomorphism.

#### Blocks

Cutpoints, bridges and blocks. Block graphs and cutpoint graphs.

#### Trees

Introduction to trees and characterizations, Applications of Trees, Spanning Trees, Minimum Spanning Trees, Trees in computer science, Centres and centroids, Blockcutpoint trees, Independent cycles and cocycles, Matroids.

#### Connectivity

Connectivity and line-connectivity, Graphical version of Menger's theorem. Traversability: Eulerian Graphs, Hamiltonian Graphs.

#### Coverings and Matching

Coverings and independence, Critical points and lines, Matching, Maximum Matching Problem, Minimum covering problems.

#### Representing Graphs

Adjacency matrix, Incidence matrix, Cycle matrix.



## **Planarity**

Plane and planar graphs, Outerplanar graphs, Kuratowski's theorem, other characterizations of planar graphs.

## **Colourability**

Vertex colouring, Chromatic number, Edge colouring, Five colour theorem, Four colour conjecture, Unique colourable graphs.

## **Directed Graphs**

Basic definitions, Type of Connectedness, Covers and Bases, Distance concepts and matrices, Connectivity, Acyclic digraphs, Cycles and traversability, Orientations and Tournaments.

## **Network Flows**

Max Flow – Min Cut Theorem, Menger's Theorem.

## **Reference Books**

- a) *F. Harary, Graph Theory, Narosa Publishing House, 1993.*
- b) *Douglas B., Introduction to Graph Theory, West, Prentice-Hall of India Pvt. Ltd., New Delhi 1999.*
- c) *NarsinghDeo, Graph Theory with Applications to Engineering and Computer Science, Prentice-Hall of India Pvt.Ltd., New Delhi, 1997.*
- d) *K.R. Parthasarathi, Basic Graph Theory, Tata McGraw-Hill Publ. Co. Ltd., New Delhi, 1994.*

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**M. Sc. 4<sup>th</sup> Semester**

**[Elective]**

**Theory of Computation**

## **Introduction**

Introduction to language theory, Alphabets, definition of phrase structure grammar, Production rules, sentences, sentential forms, language definitions, derivations.

## **Regular Languages**

Definition, Pumping Lemma of regular sets, Chomsky Hierarchy of languages. Iteration theorems. Recursive and recursively enumerable sets models, Regular languages models: finite state machines (deterministic and non-deterministic), regular grammars, regular expressions, equivalence of deterministic and non-deterministic machines and of the three models. Properties: closure, decidability.

## **Finite Automata**

Finite automaton, Deterministic, Non-Deterministic and their equivalence, Equivalence of regular expressions and FA. Moore and Mealy machines.

## **Context -Free Languages**

Relations between classes of languages, Context Free Grammar, Derivation trees, ambiguity simplification, Normal forms, applications.

## Pushdown Automata

Pushdown automata, definitions, context free languages, construction of PDA for simple CFLs, Linear bounded automata, Properties: closure, iteration theorems, parsing.

## Turing Machines

Turing machines, Introduction to computability, Universal Turing Machines, Types of Turing Machines, Techniques for construction of Turing machines, Undesirability and Halting Problem.

## Reference Books:

- a) D. S. Malik, M. K. Sen, *Discrete mathematical structure: theory and applications*, Thomson, Australia, 2004.
- b) K. P. L. Mishra, N. Chandrasekaran, *Theory of Computer Science*, Prentice Hall of India, New Delhi, 2001.
- c) J. E. Hopcroft, J.D. Ullman, *Introduction to Automata Theory, Language and computing*, Norasa Publishing, New Delhi, 2000.

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## M. Sc. 4<sup>th</sup> Semester

[Elective]

### Advanced Topology

Nets, Filters, Ultra filters, Convergence of nets and filters with special reference to zero-set filters and ultrafilters, Compact sets, Compact spaces, Alexander's theory, Tychonoff's theory, Locally compact spaces, Tychonoff's spaces, Topological embedding, Embedding Lemma, Embedding theory, Compactification, one point compactification. Alexanderoff one point compactification, Stone-Cech compactification, Ordering of compactification, minimality of  $X$  and maximality of  $\beta(X)$ .

Weak topology and completely regular topology, Topology generated by a family of pseudometrics. Uniform spaces, total boundedness, Cauchy nets and filters, Completeness, Uniformly continuous maps, Uniform isomorphism, Product uniform space, Quasi uniform spaces, Proximity spaces, Induced topology, Proximal neighbourhoods, Quasiproximity spaces.

## Reference Books

- a) K.D. Joshi, *Introduction to General Topology*, New Age International, New Delhi, 2017.
- b) J.R. Munkres, *Topology*, 2nd Ed., Pearson Education India, 2002.
- c) N. Bourbaki, *General Topology Part-I (Transl.)*, Addison Wesley, Reading, 1989.
- d) J. G. Hocking, C. S. Young, *Topology*, Addison-Wesley, Reading, 2012.
- e) J. Dugundji, *Topology*, Allyn and Bacon, Inc, 1966.
- f) J.L. Kelley, *General Topology*, Van Nostrand, Princeton, 1955.
- g) M.G. Murdeswar, *General Topology*, New Age International, 1990.
- h) S. T. Hu, *Elements of General Topology*, Holden-Day, San Francisco, 1964.
- i) S. Willard, *General Topology*, Addison Wesley, Reading Mass, 2004.

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## M. Sc. 4<sup>th</sup> Semester

[Elective]

### Fuzzy Mathematics

#### Fuzzy Relations

Introduction to fuzzy set, Fuzzy relations on fuzzy sets, composition of fuzzy relations, types of fuzzy relations, similarity methods, fuzzy graphs.

#### Fuzzy Functions

Fuzzy functions on fuzzy sets, image and inverse image of fuzzy sets, integration and differentiation of fuzzy functions.

#### Fuzzy Matrix

Sum, multiplication of two fuzzy matrices, idempotent fuzzy matrix and their problems.

#### Fuzzy Rough Set

Introduction to rough set, fuzzy rough set, soft sets and application.

#### Fuzzy Topology

Chang's definition and Lowen's definition, basic concepts, fuzzy open sets, fuzzy closed sets, fuzzy interior & fuzzy closure, fuzzy continuous function, lower (upper) semi continuous functions, their basic properties, subspaces, product spaces, quotient spaces, intuitionistic fuzzy topological spaces, Induced fuzzy topology, Separation axioms in fuzzy topological spaces, Fuzzy filter and fuzzy net, Fuzzy compact spaces, Fuzzy connected space and fuzzy countability axioms.

#### Reference Books

- a) H.J Zimmermann, *Fuzzy Set Theory and Its Applications*, Allied Publisher's Limited, 2014.
- b) Robert Babuška, *Fuzzy Modelling for Control*, International Series in Intelligent Technologies, Volume 12, 2012.
- c) N. Palaniappan, *Fuzzy Topology*, Norosa, 2006.

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## M. Sc. 4<sup>th</sup> Semester

[Elective]

### Decision Theory and Computational Statistics

Games and statistical games, Statistical decision problem, Decision function, Risk function, Prior and posterior distribution, Baye's risk and Baye's rules.

Least favourable prior, Minimacity, Admissibility and complete classes, Admissibility of Baye's rules, Existence of minimal complete class and Baye's rules.

The Supporting and Separating Hyperplane theorems, Essential completeness of the class of nonrandomized rules, Minimax and complete class theorems.

Solving for minimax rules, Essential completeness of class of rules based on sufficient statistics, Continuity of risk function, Invariant decision problems, Admissible and minimax invariant decision rules. Analysis of variance, one- way and two – way classification.

Concept of design of experiment, Some standard design: Completely randomized design, randomized block design, Latin Squares, Graeco Latin Squares and Factorial Design.

Confounding and blocking in Factorial design, fractional factorial design. Simple and multiple regression models.

Classical techniques of Time series analysis, Smoothing and decomposition, Analysis of covariance model.

### Reference Books

- a) *James O. Berger, Statistical Decision Theory, Foundation, Concepts and Methods, Springer series in Statistics, 1980.*
- b) *J.E. Gentle and Wolfgang HSrdle, Yuichi Mori, Handbook of computational statistics: Concepts and methods, 2004.*
- c) *James O Berger, Statistical Decision Theory and Bayesian Analysis, Springer series, 2<sup>nd</sup> edition, 1993.*

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**M. Sc. 4<sup>th</sup> Semester**

**[Elective]**

**Financial Mathematics**

### Introduction

Some Basic definitions and terminology.

### Basic Theory of Option Pricing

Single and Multi-Period Binomial Pricing Models, Cox-RossRubinstein (CRR) Model, Black Scholes Formula for Option Pricing as a Limit of CRR Model.

### Stochastic Calculus

Brownian and Geometric Brownian Motion, Theory of Martingales. Stochastic Calculus, Stochastic Differential Equations, Ito's Formula to Solve SDE's. Applications of Stochastic Calculus in Option Pricing.

### Mean-Variance Portfolio Theory

Markowitz Model of Portfolio Optimization and Capital Asset Pricing Model (CAPM). Limitations of Markowitz Model and New Measures of Risk.

## **Interest Rates and Interest Rate Derivatives**

Binomial Lattice Model, Vasicek, Hull and White Models for Bond Pricing.

### **Reference Books**

- a) *D. G. Luenberger, Investment Science, Oxford University Press, 2013.*
- b) *M. Capiński, T. Zastawniak, Mathematics for Finance: An Introduction to Financial Engineering, Springer, 2010.*
- c) *Thomas Mikosch, Elementary Stochastic Calculus with Finance in view, World Scientific, 1998.*
- d) *Suresh Chandra, S. Dharmaraja, Aparna Mehra, R. Khemchandani, Financial Mathematics: An Introduction, Narosa Publishing House, 2013.*
- e) *S. E. Shreve, Stochastic Calculus for Finance, Vol. I & Vol. II, Springer, 2010.*
- f) *Sean Dineen, Probability Theory in Finance: A Mathematical Guide to the Black-Scholes Formula, American Mathematical Society, Indian edition, 2013.*

### **M. Sc. 4<sup>th</sup> Semester**

**Syllabus for industrial project will be specified by the respective industry.**

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